TOPOLOGICAL DESIGN OF PROTOCOLS

ARTHUR JAFFE, ZHENGWEI LIU, AND ALEX WOZNIAKOWSKI

Abstract. We give a topological simulation for tensor networks that we call the *two-string model*. In this approach we give a new way to design protocols, and we discover a new multipartite quantum communication protocol. We introduce the notion of topologically compressed transformations. Our new protocol can implement multiple, non-local compressed transformations among multi-parties using one multipartite resource state.

1. Introduction

Manin and Feynman introduced the concept of quantum simulation for quantum systems [1, 2, 3]. Here we give a *topological simulation* for a quantum process by using a topological model. This leads us to a new way to design protocols, using isotopy. In our work the concept of isotopy differs from the usual topological isotopy of diagrams, as our diagrams may have charge. We call the corresponding property that they satisfy *para-isotopy*.

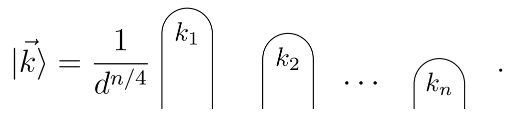
We use a *two-string model* of topological simulation. This means that we represent a one-qudit transformation (such at the Pauli matrices X, Y, Z) by diagrams with two input points and two output points. We use this model to introduce the concept of *topologically compressed* transformations of qudits. We focus much of this paper on illustrating the usefulness of our method by discussing a particular example. We use topological simulation to design a new diagrammatic protocol that we call *multipartite compressed teleportation* (MCT). We apply this protocol to implement non-local compressed transformations—using optimized resource states, local transformations, and classical communication (LOCC). We show how one can represent MCT in terms of the usual algebraic elements that one employs in circuit design.

To implement the translation between diagrammatic protocols and algebraic protocols, we use a bi-directional dictionary that relates the two terminologies. We give here elements of this dictionary that one needs for this paper and refer the reader to our companion paper [4] for additional details, as well as to [5] for mathematical foundations. Let us explain our main concepts one by one.

2. The Main Concepts

2.1. **Topological Simulation and the Two-String Model.** We represent the concepts in quantum information by charged strings. Let d be the dimension of the single qudit space.

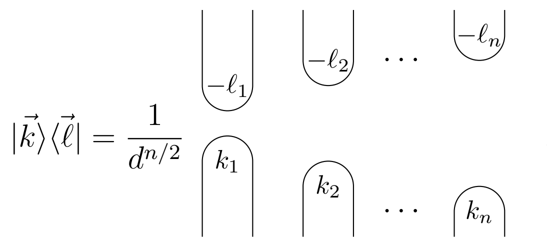
We represent the n-qudit basis $|\vec{k}\rangle = |k_1, k_2, \cdots, k_n\rangle$ as n charged caps:



(1)

Here the charges are labelled by $0 \leqslant k_j < d$, and charges add modulo d. One can omit the label for charge zero. Since n-qudits have $2n$ output points, we call this a two-string model.

One denotes the adjoint by a charge-inverting vertical reflection, so the n-qudit matrix units $|\vec{k}\rangle\langle\vec{\ell}| = |k_1, k_2, \cdots, k_n\rangle\langle \ell_1, \ell_2, \cdots, \ell_n|$ are represented by the diagrams



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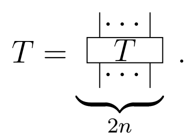
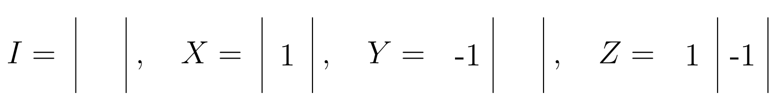
In general an n-qudit transformation T is a diagram with $2n$ input points on the top and $2n$ output points on the bottom,



(3)

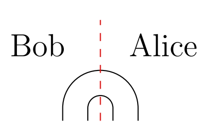
The algebraic rules for computation are captured by the diagrammatic relations for charged strings in Appendix A.

2.1.1. **Pauli Matrices.** To make clear the usefulness of such a representation, we mention that certain transformations which form the basis for protocols can be expressed by elementary diagrams. In particular, the 1-qudit identity transformation I, and the Pauli matrices X, Y, Z, are represented by the elementary diagrams



(4)

2.1.2. **Bell State.** Moreover the Bell state, as a two-qudit resource state shared by Alice and Bob, is $d^{-1/2}$ times



(5)

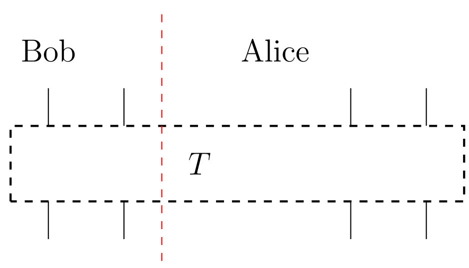
Here only the double caps represent the Bell state; the other labels are for explanation. The dashed, red line indicates that the two persons have distinct localizations. The double cap can pass the red line. This means that the corresponding state can be shared between Alice and Bob as an entangled resource state. We use a corresponding n-qudit resource state given in (7) for designing our protocol.

2.2. **Teleportation.** One could say that the modern theory of quantum communication networks began in 1993 with the teleportation protocol discovered by Bennett, Brassard, Crépeau, Jozsa, Peres, and Wooters [6]. This protocol allows one to disassemble a quantum state located at Alice's location, and to reconstruct it at Bob's location. In order for the reconstruction to succeed, Alice and Bob prearrange to share a specific entangled state, which is utilized as a resource for the protocol. In addition, they share some purely classical information.

Preskill, Gottesman, and Chuang described the notion of quantum software for solving problems in quantum computation and quantum communication [7, 8]. Recently, Priandola and Braunstein cite teleportation as the "most promising mechanism for a future quantum internet" [9]. One can realize quantum networks through bidirectional quantum teleportation (BQST). Experimental work on long-distance teleportation has been achieved [10, 11, 12]. The *Quantum Science Satellite* built by Pan and his coworkers provides an opportunity to test teleportation at record-breaking distances [13, 14].

2.3. **Topological Compression.** Our notion of topological compression becomes transparent in terms of the two-string model for quantum information, for one can visualize compression of a transformation in terms of the diagrams that describe it. Very roughly, the information for a compressed transformation is carried by one of the two strings. We explain this in terms of an example.

Suppose that Alice and Bob are at separate locations and want to implement a non-local, two-qudit transformation T. The topological simulation of that goal is given by the following diagram:



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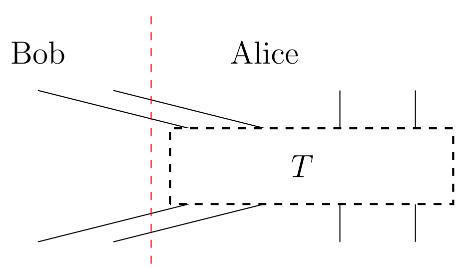
If one applies a topological isotopy, one can move the red line so the transformation is performed completely by Alice. This is the solution, and its topological simulation is



. (6)

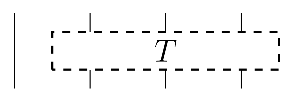
This topological isotopy does not change the function of the diagram, but does change its interpretation in quantum information. After isotopy, the diagram means that Bob can teleport his input to Alice; then Alice can implement the transformation T locally on her computer and teleport the result back to Bob using BQST.

It is well-known that the cost of teleportation for a general transformation is two resource states. Recall that one resource state allows two strings to pass across the red dashed line, as explained for the Bell state in (5). Thus one has an indication from (6) that the cost of teleportation can be estimated by counting the number of strings that pass over the red dashed line.

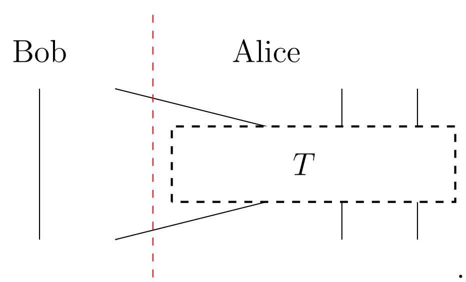
However Zhou et al. and Eisert et al. pointed out that the cost of BQST may not be optimal. For certain transformations, including CNOT, they gave a teleportation protocol with lower cost [15, 16]. This optimization has been further studied in [17], and in [18, 19] one finds extensive references.

So it is natural to ask the question: what transformations can be teleported with less cost, compared with BQST? We now characterize *topologically compressed transformations* and show that they have this property. All controlled transformations are topologically compressed.

Following the above discussion, consider any 2-qudit transformation that can be represented by the following diagram:



In other words, such a transformation acting on Bob's qudit only requires the information on one of the two strings. We say that such transformations are topologically compressed; we give the precise statement as Definition 3.1. This covers a large variety of transformations that are commonly used in protocols. In this case the corresponding isotopy yields:



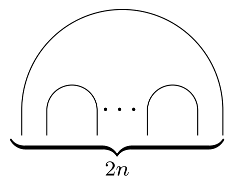
As only two strings pass over the red line, one expects that it is possible to implement this non-local transformation using only one resource state, rather than two.

In §2.4, we introduce a new protocol to teleport information that is captured on one string using one resource state. We call this protocol *compressed teleportation* (CT). It relies on using local operations and classical communication (LOCC). And it optimizes the entanglement resource cost for teleportation of compressed transformations. The CT protocol reduces the costs by 50% compared with BSQT. In BSQT, one needs two resource states. In our CT protocol, we use one resource state. Since CNOT, Tofolli, and controlled transformations are all topologically compressed, our protocol covers the previous teleportation protocols for CNOT, Tofolli, and controlled transformations.

2.4. **Multipartite Communication.** Greenberger, Horne, and Zeilinger introduced the classic multipartite resource state that we denote $|\mathrm{GHZ}\rangle$ in [20]. Experimental work on $|\mathrm{GHZ}\rangle$ was achieved in [21, 22, 23].

We introduce our n-qudit resource state $|\mathrm{Max}\rangle$ in our companion paper [4]. This state generalizes the Bell state (5) and has the diagrammatic representation,

$$|\mathrm{Max}\rangle = d^{-n/4}$$



. (7)

The resource states $|\mathrm{Max}\rangle$ and $|\mathrm{GHZ}\rangle$ are related by the quantum Fourier transform (14), as we show in [4].

Topological compression is compatible with the topological feature of our multipartite resource state $|\mathrm{Max}\rangle$. This allows one to generalize our CT protocol to multipartite communication, with the cost of one multipartite resource state, and we call this generalization multipartite compressed teleportation.

Our MCT protocol does not reduce to multiple, bipartite communications. If one realizes this teleportation by BQST, then one would need n bipartite resource states, and constructing these requires $2n$ noiseless channels. In our MCT protocol, we use one n-partite resource, which requires n noiseless channels to construct. Therefore we reduce the cost by 50%.

3. The MCT Protocol

3.1. **1-qudit transformations.** Recall that d is the dimension of the 1-qudit space, $q = e^{\frac{2\pi i}{d}}$, and $\zeta = q^{1/2}$ is a square root of q satisfying $\zeta^{d^2} = 1$. The matrices X, Y, Z, F, G play an important role. Three of these are the qudit Pauli matrices

$$X|k\rangle = |k+1\rangle\ ,\ Y|k\rangle = \zeta^{1-2k}|k-1\rangle\ ,\ Z|k\rangle = q^k|k\rangle\ . \tag{8}$$

The quantum Fourier transform matrix F and the Gaussian matrix G are defined by

$$F|k\rangle = \frac{1}{\sqrt{d}}\sum_{\ell=0}^{d-1} q^{k\ell}|\ell\rangle\ , \quad G|k\rangle = \zeta^{k^2}|k\rangle\ . \tag{9}$$

These matrices are unitary, $X^d = Y^d = Z^d = F^4 = G^{2d} = I$, and they satisfy many interesting relations, among them

$$XY = qYX\ , \quad YZ = qZY\ , \quad ZX = qXZ\ , \tag{10}$$

$$XYZ = \zeta\ , \quad FXF^{-1} = Z\ , \quad GXG^{-1} = Y^{-1}\ . \tag{11}$$

3.2. **Resource States.** The 1-qudits with d possible values of the charge can be considered elements of the group $\mathbb{Z}_d$, and for an n-qudit $|\vec{k}\rangle = |k_1, k_2, \cdots, k_n\rangle$, let $|\vec{k}| = \sum_{j=1}^{n} k_j \in \mathbb{Z}_d$ denote the total charge. In the companion paper [4] we introduced the multipartite entangled resource state

$$|\mathrm{Max}\rangle = d^{\frac{1-n}{2}} \sum_{|\vec{k}|=0} |\vec{k}\rangle \ . \tag{12}$$

On the other hand, the $|\mathrm{GHZ}\rangle$ resource state is equal to a different sum,

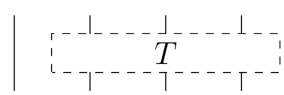
$$|\mathrm{GHZ}\rangle = d^{-\frac{1}{2}} \sum_{l=0}^{d-1} |k, k, \cdots, k\rangle \ . \tag{13}$$

In fact these two resource states are related by the local operation

$$|\mathrm{GHZ}\rangle = (F \otimes \cdots \otimes F)|\mathrm{Max}\rangle \ , \tag{14}$$

where F denotes 1-qudit quantum Fourier transform defined in (9).

3.3. **Topologically Compressed transformations.** If the diagrammatic representation of a 2-qudit transformation T has a free through string on the left,



then we consider such transformations as topologically compressed.

We give algebraic characterizations of such transformations. A transformation has such a representation if and only if it commutes with the action of Pauli X on the first qudit.

In general, we say that a transformation T is X-compressed on the i^{th}-qudit if T commutes with the action of Pauli X on the i^{th}-qudit. Similarly we say T is Y (or Z)-compressed on the i^{th} qudit, if it commutes with the action of Pauli Y (or Z) on the i^{th} qudit. We can switch between the three compressed transformations using $FXF^{-1} = Z$ and $GXG^{-1} = Y^{-1}$.

Note that a transformation T is Z-compressed on the first qudit if and only if T is

$$T = \sum_{\ell=0}^{d-1} |\ell\rangle\langle\ell| \otimes T(\ell) \ , \tag{15}$$

where the $T(\ell)$'s are transformations on the remaining qudits. That means T is a controlled transformation, or a block diagonal transformation.

Definition 3.1. *The transformation T' is compressed on the i^{th}-qudit if $T' = UTV$, where T is Z-compressed on the i^{th}-qudit and U, V are local transformations on the i^{th}-qudit.*

3.4. **MCT for Controlled Transformations.** Suppose a network has one leader and n parties. Also assume that the j^{th} party can perform a controlled transformation

$$T_j = \sum_{l=0}^{d-1} |\ell\rangle\langle\ell| \otimes T_j(\ell), \tag{16}$$

where the control qudit belongs to the person P_j in the j^{th} party, and $T_j(\ell)$ can be an arbitrary multi-person, multi-qudit transformation on the targets. (The notation for the controlled transformation T_j is shown in Figure 1.)

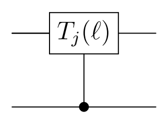
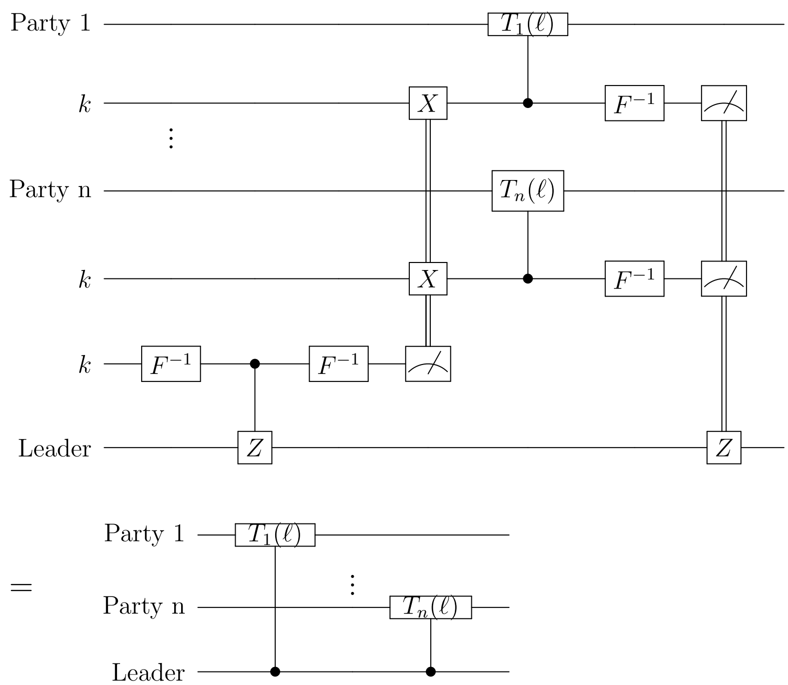


FIGURE 1. Controlled transformations.

With these assumptions, we design a circuit shown on the left of Equation (18), so that the leader can perform a controlled transformation T_c to all parties in the network,

$$T_c = \sum_{l=0}^{d-1} |\ell\rangle\langle\ell| \otimes T_n(\ell) \otimes \cdots \otimes T_1(\ell) \ . \tag{17}$$

This requires using a multipartite resource state $|\text{Max}\rangle$ among the leader and the persons P_j. The function of the circuit is shown on the right of Equation (18). The leader has the common control qudit in T_c, and the j^{th} party performs the transformation $T_j(\ell)$ for control qudit ℓ. This protocol costs one resource state $|\text{Max}\rangle$ and $2n$ cdits (classical information channels). The time cost is the transmission of two cdits and the implementation of local transformations.



(18)

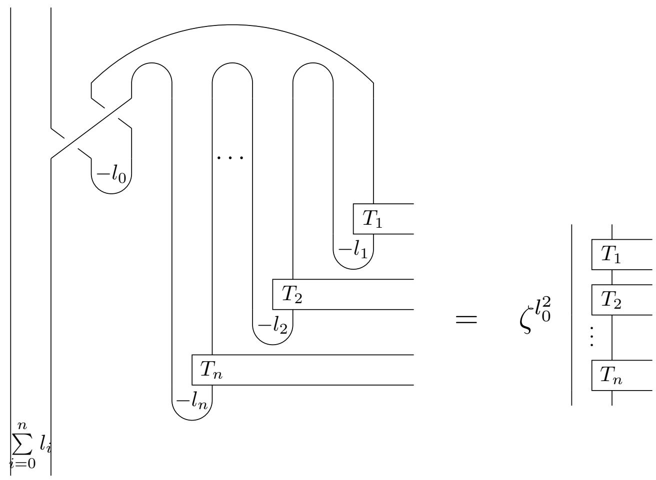
In the MCT algebraic protocol, the k's arise from the $|\mathrm{GHZ}\rangle$ resource state (13) for the leader and the parties: Party j, where $1 \leqslant j \leqslant n$. The output of the protocol is the multipartite, controlled transformation.

We specify the MCT in the usual algebraic terminology as a circuit. One can consider CT a special case for two parties. From this picture, one can understand the protocol without knowing its topological significance. In §4 we derive this protocol from topological simulation.

4. Topological simulation for MCT

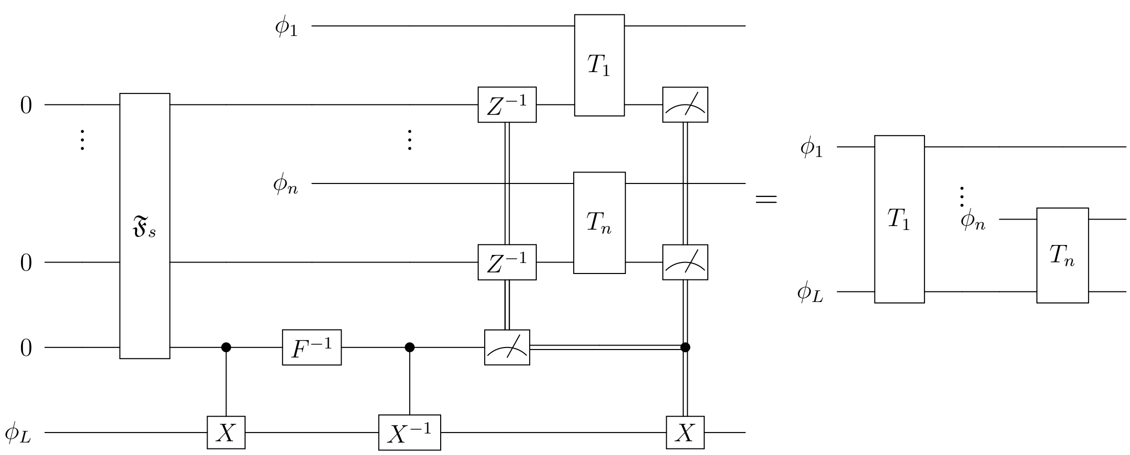
We give the MCT diagrammatic protocol for X-compressed transformations. The design of this protocol is equivalent to the design for Z-compressed transformations by applying unitary conjugation.

First we simulate the goal using the picture on the right side in Equation (19). That means the leader wants to share X-compressed transformations with n parties. Then we apply topological isotopy and we obtain the picture on the left after adding charges which indicate the results of the measurements and recovery maps given by Pauli X. Finally we obtain the picture on the left which is a diagrammatic protocol for MTC. It includes one multipartite resource state $|\text{Max}\rangle$ and LOCC.



(19)

One can use the dictionary to translate from the diagrammatic protocol to the algebraic circuit in (20).



(20)

The state $|\vec{0}\rangle$ denotes the n-qudit with charge 0 for each 1-qudit, and we call $|\vec{0}\rangle$ the ground state. It plays the role of ancilla qudits in our protocol. We mention the extremely interesting transformation $\mathfrak{F}_s$ that appears here, and that we call the *string Fourier transform*. We explore $\mathfrak{F}_s$ extensively in [5], and we show in [4] that $|\text{Max}\rangle = \mathfrak{F}_s|\vec{0}\rangle$, which replaces $|\text{Max}\rangle = (F^{-1} \otimes \cdots \otimes F^{-1})|\text{GHZ}\rangle$ given by (14).

One can simplify the protocol by the identity in Figure 2.

Taking the conjugation of local transformations, we obtain the MCT protocol for other types of compressed transformations. In particular, taking the conjugate of the Fourier transform F, we obtain the MCT protocol for Z-compressed transformations or controlled transformations in (18) .

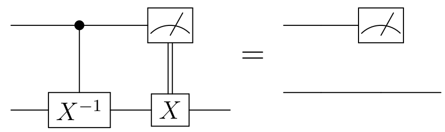


FIGURE 2.

In the case with only two persons, the MCT protocol says: Assume that a quantum network can perform a transformation T, which is compressed on a 1-qudit belonging to a network member Alice. Then Alice can teleport her 1-qudit transformation to Bob using one edit and two cdits. One can easily derive the entanglement-swapping protocol, and the teleportation of the Tofolli gate from it.

5. Conclusion

In this paper we investigate a two-string model for quantum information.

5.1. **What is new in our method?**

(1) We give a topological design for protocols and obtain a new protocol for multipartite communication.
(2) Our two-string model for one-qudit transformations allows us to introduce the concept: topological compression.
(3) Our diagrams have charged strings, unlike previous models, and require the notion of paraisotopy.
(4) We represent a qudit basis, measurements, and Pauli matrices as charged strings, so that we can design protocols in a topological manner.
(5) Our new protocol costs only one multipartite resource state to implement multiple, non-local transformations between multiple parties.
(6) For more than two parties, our multipartite teleportation protocol does not reduce to compositions of bipartite communications.

In [4] we analyze the BVK protocol [24] using holographic software. It would be interesting to analyze other protocols by this method, such as those in [25, 26, 27, 8, 28, 29, 30, 31, 32, 33].

6. Acknowledgements

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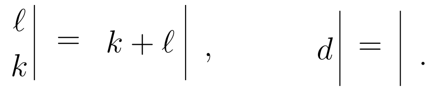
Appendix A. Diagrammatic Relations

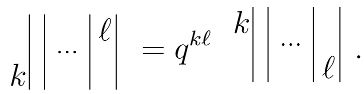
A.1. **Planar relations.** In this section we give relations between certain diagrams. The consistency of these relations is proved in [5]. Using these relations, we give a dictionary between qudits, transformations, measurements, and diagrams.

A.1.1. *Addition of charge, and charge order.*



(21)

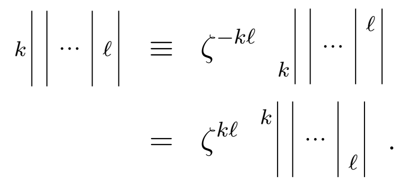
A.1.2. *Para-isotopy.*



(22)

Here the strings between charge-k string and charge-ℓ string are not charged. We call $q^{k\ell}$ the twisting scalar. Let $\zeta = q^{1/2}$, such that $\zeta^{d^2} = 1$.

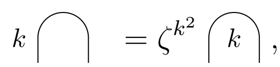
Notation: The twisted tensor product of pairs interpolates between the two vertical orders of the product. In the twisted product, we write the labels at the same vertical height:



(23)

In this case $k, l \in \mathbb{Z}$, and k and $k+d$ yield different diagrams. If the pair is neutral, namely $\ell = -k$, then the twisted tensor product is defined for $k \in \mathbb{Z}_d$.

A.1.3. *String Fourier relation.*



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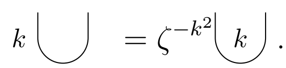
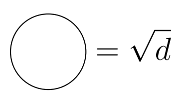


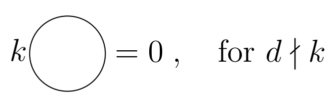
(25)

A.1.4. *Quantum dimension.*



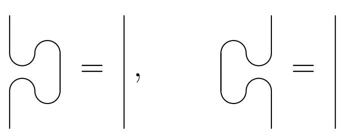
(26)

A.1.5. *Neutrality.*



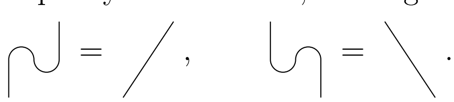
(27)

A.1.6. *Temperley-Lieb relation.*



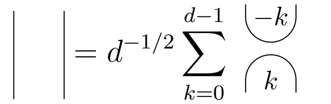
(28)

Notation: Based on the Temperley-Lieb relation, a string only depends on the end points:



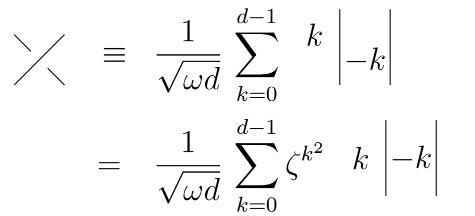
(29)

A.1.7. *Resolution of the identity.*



(30)

A.1.8. *Braid.* With $\omega = \frac{1}{\sqrt{d}}\sum_{j=0}^{d-1}\zeta^{j^2}$, note that $|\omega| = 1$. The positive braid is



(31)

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