Multiple Reflections

ETH

23 May, 2016

Three Stages of Reflection

- Some reflections on Robert
- Some reflections on reflection positivity
- Some reflections after Robert

Reflections on Robert

- I first met Robert 48 years ago in May 1968
  - I was guest professor at ETH, courtesy Jost and Hepp
  - Robert was working on his thesis, completed 1969
  - Konrad Osterwalder was my assistant



Robert



Konrad

Seminar für Theoretische Physik

Hochstrasse 60





Zürich

Reflections on Robert

- Robert came to Harvard as my first post-doctoral fellow in the fall of 1970 (easy to arrange)
- We spent the spring 1971 together in Princeton.
- First project was infinite volume limit of Yukawa$_2$.
- Konrad joined us in fall 1971 at Harvard as post-d.

Life at Harvard in 1970

- Robert lived on Huron Avenue in “west Cambridge.”
- He became good friends with the particle physicists: Shelly Glashow, Sidney Coleman, and John Iliopolous (time of GIM mechanism).
- Many unmarried faculty: Coleman, Glashow, AJ. Faculty and postdoctoral fellows socialized a lot, occasionally also Callan, Gross, Martin, Glauber, Schwinger.
- Regular Sunday lunch at the Peking on Mystic.

Life at Harvard in 1970's

- Robert led an unconventional life.
- His friend Christa, whom he met on a skiing trip, was East German. So Robert often went to visit.
- The Stasi attempted to recruit Robert as a spy, interrogating him and threatening her family.
- Robert wanted to get Christa to the West legally. I got into the middle of that mess, the FBI got involved, and so did the German government.
- This took 5 years, 2 kids, and a lot of money.
- In 1992, after the wall came down, I gave a talk in Berlin. Robert took me to see the prison (then a museum) where he had been interrogated some 20 year before.

Reflections on Reflection Positivity

On the Discovery

- Summer 1972, Robert had gone on vacation to visit Christa.
- I began discussing with Konrad the paper of Edward Nelson on Markov field reconstruction.
- I was interested to know if there was a basic principle that worked independent of the Markov property, and might apply to other situations.
- Then Robert came back from his vacation.
- He had left his car in Northampton, where I had an apartment. There one could park on the street without limit of time.
- I drove Robert to pick up his car. We had a very interesting 2-hour trip!
- I left from Nothampton to Chicago for a conference.
- When I returned to Boston 4 days later, O&S had discovered RP!
- Writing it up took almost two years.
- I was deeply tied up in my work on cluster expansions and on three dimensions.

Reflections on Reflection Positivity

What is RP?

– Basic Schrödinger Equation:

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

– Hamiltonian (Energy). Want stability:

$$H \geqslant 0$$

– Ensures analyticity in a complex half t-plane.

Modify Equation: Continue analytically in t

Heat Diffusion Equation:

$$-\hbar \frac{\partial \psi}{\partial t} = H\psi$$

Imaginary (unphysical) time:

$$t \rightarrow -it$$

Easier to study, BUT:

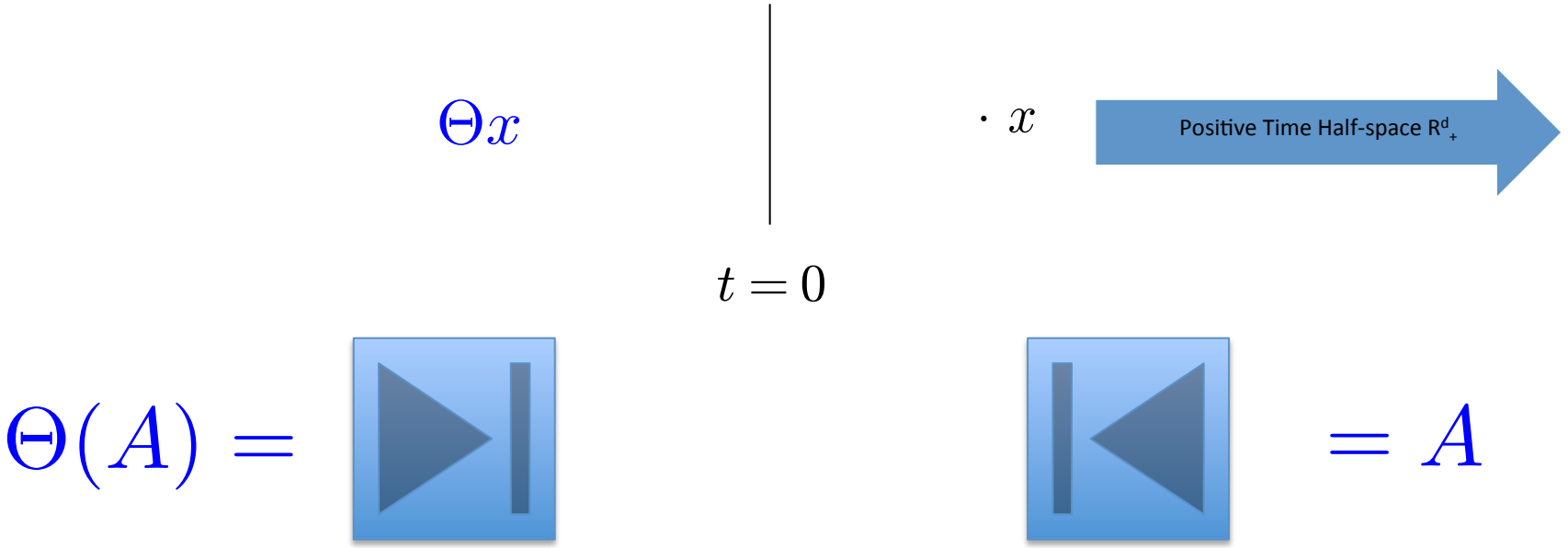
Question: How can one get back? Answer: RP.

Reflections on Reflection Positivity

What is RP?

Pictorial Interpretation

Horizontal (time) Reflection



Antilinear Isomorphism

Reflection Positivity Hypothesis

$$0 \leqslant \underbrace{\langle \Theta(A) A \rangle}_{\text{classical}} = \underbrace{\langle A,\, A \rangle_{\mathcal{H}}}_{\text{quantum}}$$

Consequence Stability (Osterwalder-Schrader)

$$0 \leqslant \underbrace{\langle \Theta(A) A(t) \rangle}_{\text{classical}} = \underbrace{\langle A,\, e^{-tH}\, A \rangle_{\mathcal{H}}}_{\text{quantum}} \leqslant \underbrace{\langle A,\, A \rangle_{\mathcal{H}}}_{\text{quantum}}$$

O-S Axioms ≅ Wightman Axioms

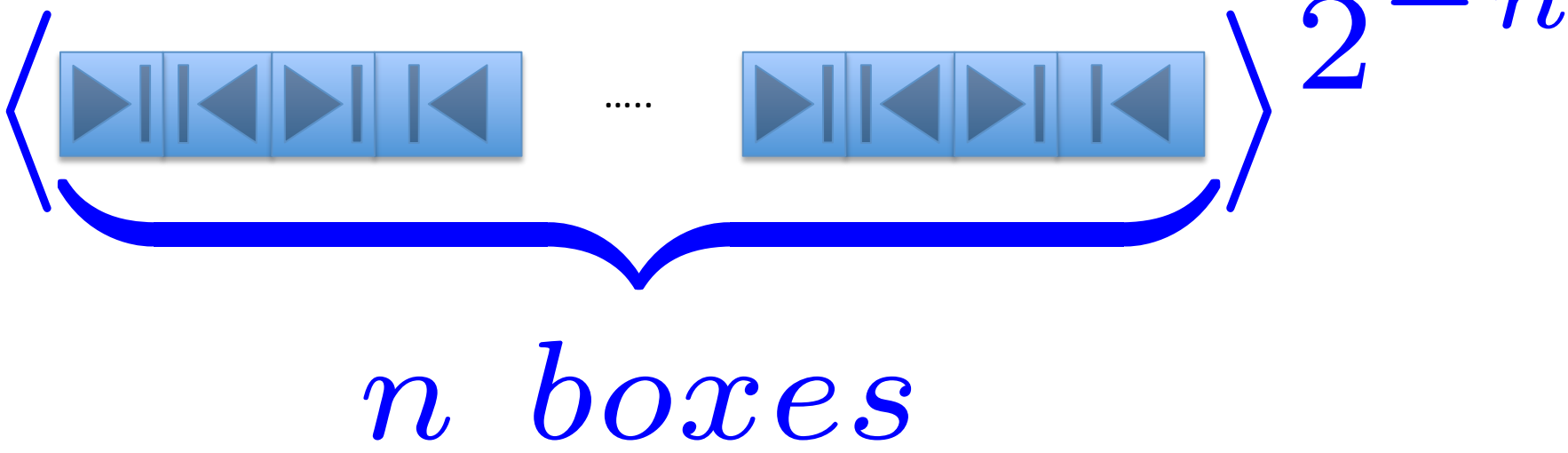
Enabled the First Non-Trivial QFT Model

- Weakly-coupled polynomial scalar fields in two dimensions. Glimm, AJ, Spencer (1974)
  - Wightman axioms for relativistic QFT
  - Unique vacuum
  - Mass gap
  - Upper gap (isolated one-particle state)

Multiple Reflection Bound

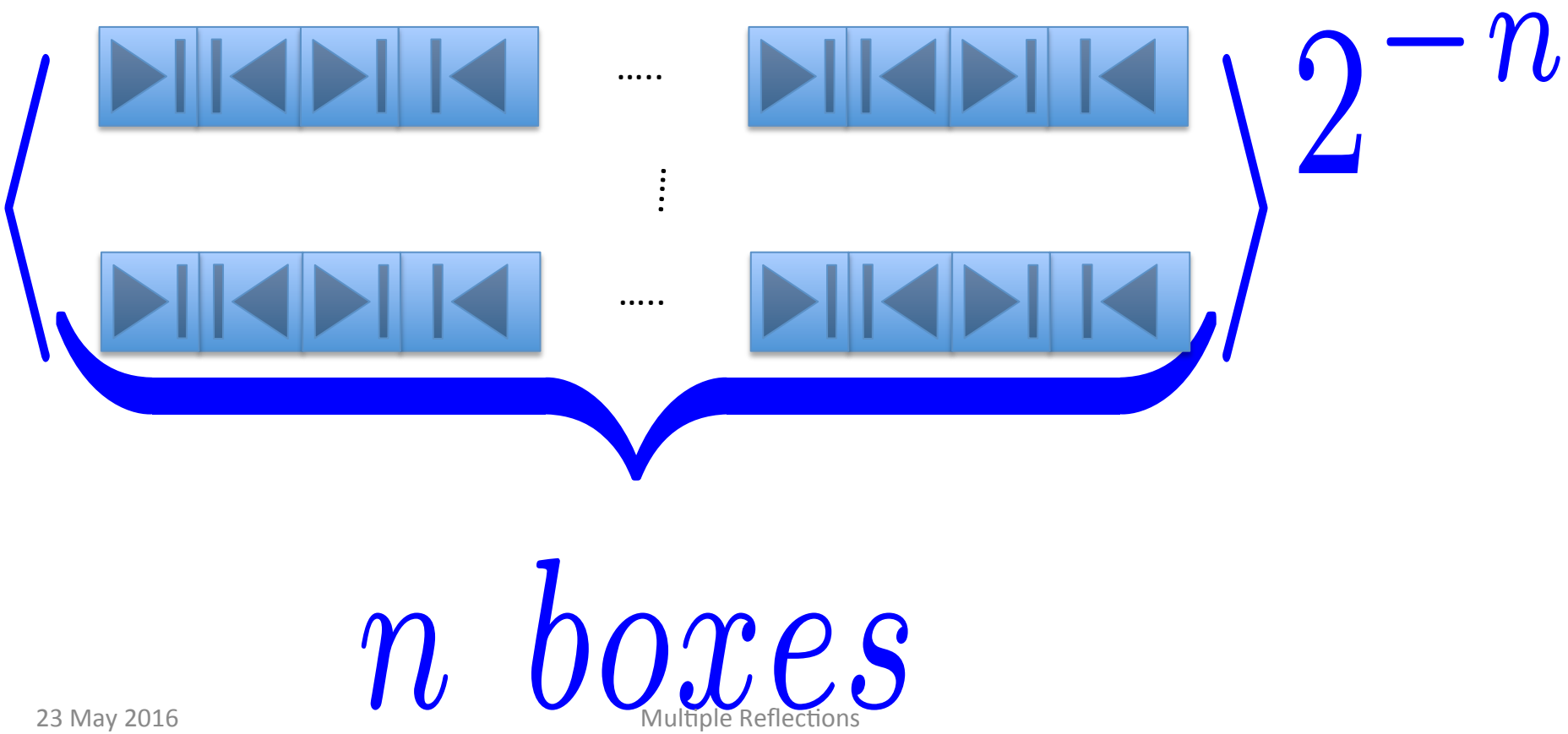
$$\begin{aligned} |\langle A \rangle| &\leqslant \langle \Theta(A) A \rangle^{1/2} \\ &= |\langle (\Theta(A))(t) A(t) \rangle|^{1/2} \end{aligned}$$

Pictorial upper bound:



Also Reflect Vertically

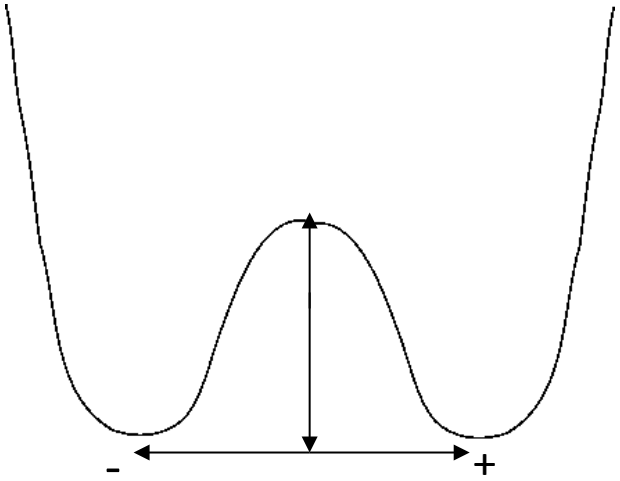
Pictorial upper bound:



Reflections on Reflection Positivity

Phase Transitions

Discrete symmetry breaking



$$V(\varphi) = \lambda^{-2}\left(\lambda^2\varphi^2 - 1\right)^2$$
$$\lambda \ll 1$$

Quantum Mechanics: Unique Node-less Ground State (Tunnelling)

Field Theory: Non-Unique Ground State (Possible Lack of Tunnelling, as in Ising model)

Dobrushin-Minlos: announcement 1973

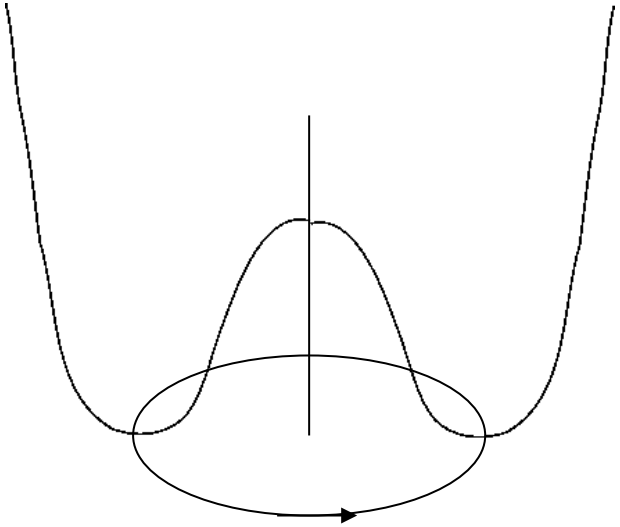
Reflections on Reflection Positivity
Phase Transitions

- First proof of phase transitions in QFT. Strongly coupled Glimm, AJ, Spencer (1975)
- Idea: use RP and multiple reflection bound to estimate deviation of mean field from "Peierls argument" studied by Dobrushin and by Griffiths. Thereby one can show this method also applies in the field case.
- Dobrushin-Minlos retraction 1976
- Wightman axioms, unique vacuum, mass gap GJS 1976

Reflections on Reflection Positivity

Phase Transitions

Continuous Symmetry Breaking



$$V(\vec{\varphi}) = \lambda \left(\vec{\varphi}^{\,2} - \tfrac{1}{\lambda}\right)^2 , \quad \vec{\varphi} \in R^N$$

Fröhlich-Simon-Spencer (1976): Beautiful "Infra-Red Bound."

$$0 \leq S(p) - c\delta(p) \leq c_1 \frac{1}{p^2}$$

d ≥ 3, N ≥ 1

integrable

Reflections on Reflection Positivity
Other Field Theories

Fermions: Osterwalder-Schrader

Osterwalder-Eckmann, Borthwick

Lattice Gauge Theory: Osterwalder-E. Seiler

Higher Spin: Ozkaynak

Complex Fields: AJ, Jäkel, Martinez

Spectral condition on a torus

Superspace: Osterwalder, (Lesniewski), AJ, in progress

Reflections on Reflection Positivity
Statistical Mechanics

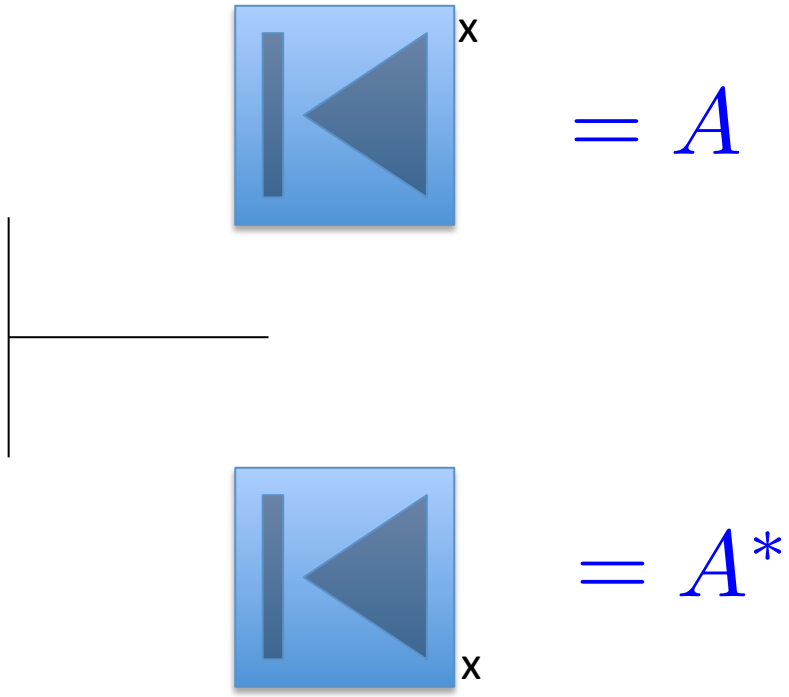
Spin Systems: Fröhlich, Dyson, Israel, Simon, Lieb, Brydges, Sokal

Characterization Majoranas: AJ-Pedrocchi, AJ-Janssens

Parafermions: AJ-Pedrocchi AJ-Janssens

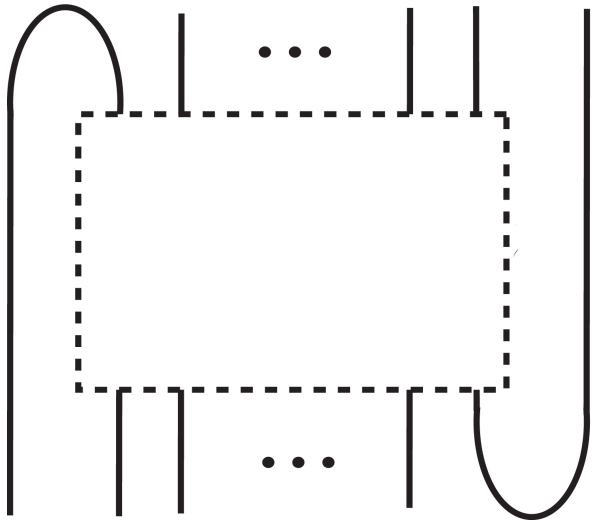
New: Geometric Interpretation of RP
(work with Zhengwei Liu)
Vertical Reflection

As anti-isomorphism
(planar algebra).
The operator adjoint.



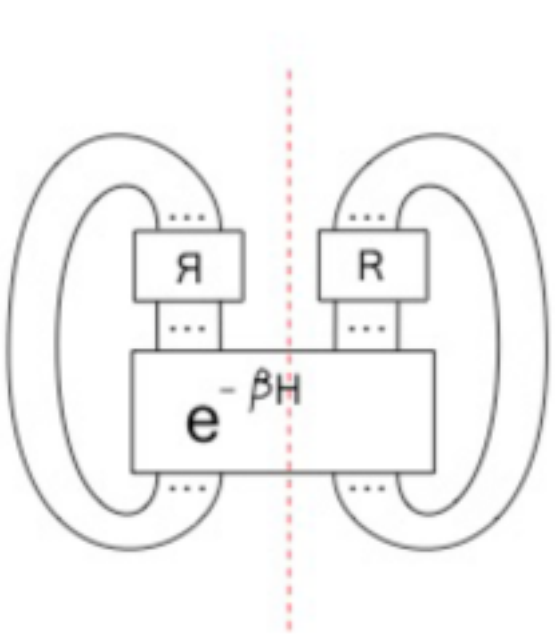
String Fourier Transform

$$(\mathfrak{F}_s T)(x_1, \ldots, x_n) = T(x_2, \ldots, x_n, x_1)$$

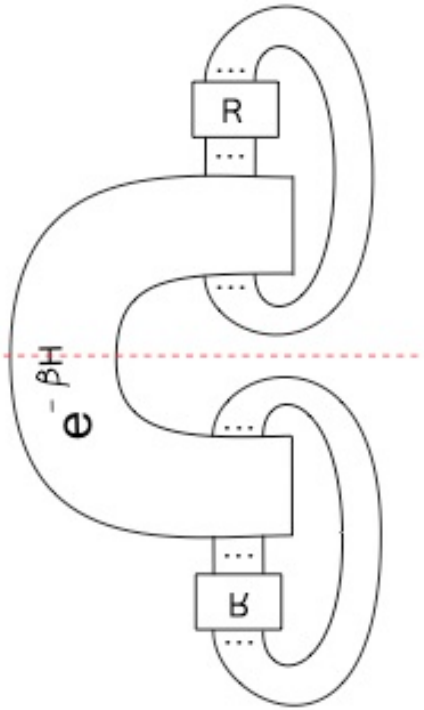


Partial Rotation

Horizontal / Vertical Relation



$=$



$= \langle A^* A \rangle \geqslant 0$

Para isotopy

Rotation + Horiz.-Reflection = Vert.-Reflection

Theorem

Consider a scalar, fermion, or parafermion statistical mechanics in a doubled von Neumann algebra (e.g. pos./neg. time). Let Θ denote the Tomita reflection. If H is a Hamilton with positive SFT, then positive temperature states satisfy RP.

AJ-Liu Planar para algebras (2016), arxiv 1602.02662

Uses twisted product:

AJ-Pedrocchi, CMP **337** (2015), 455–472

AJ-Janssens, CMP (2016)

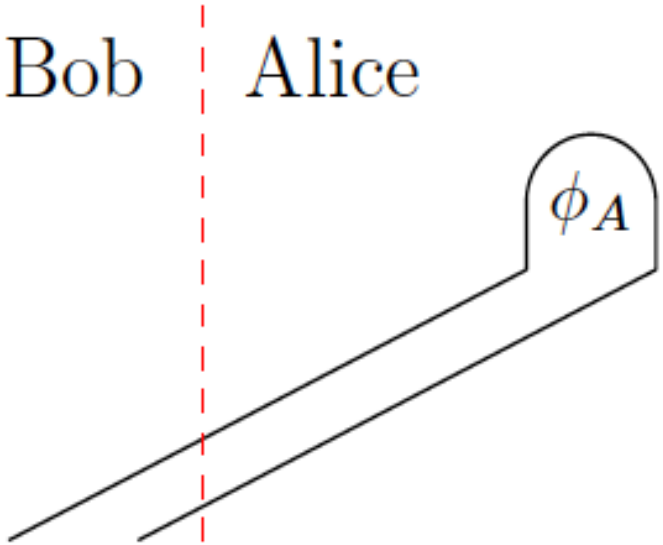
http://dx.doi.org/10.1007/s00220-015-2545-z.

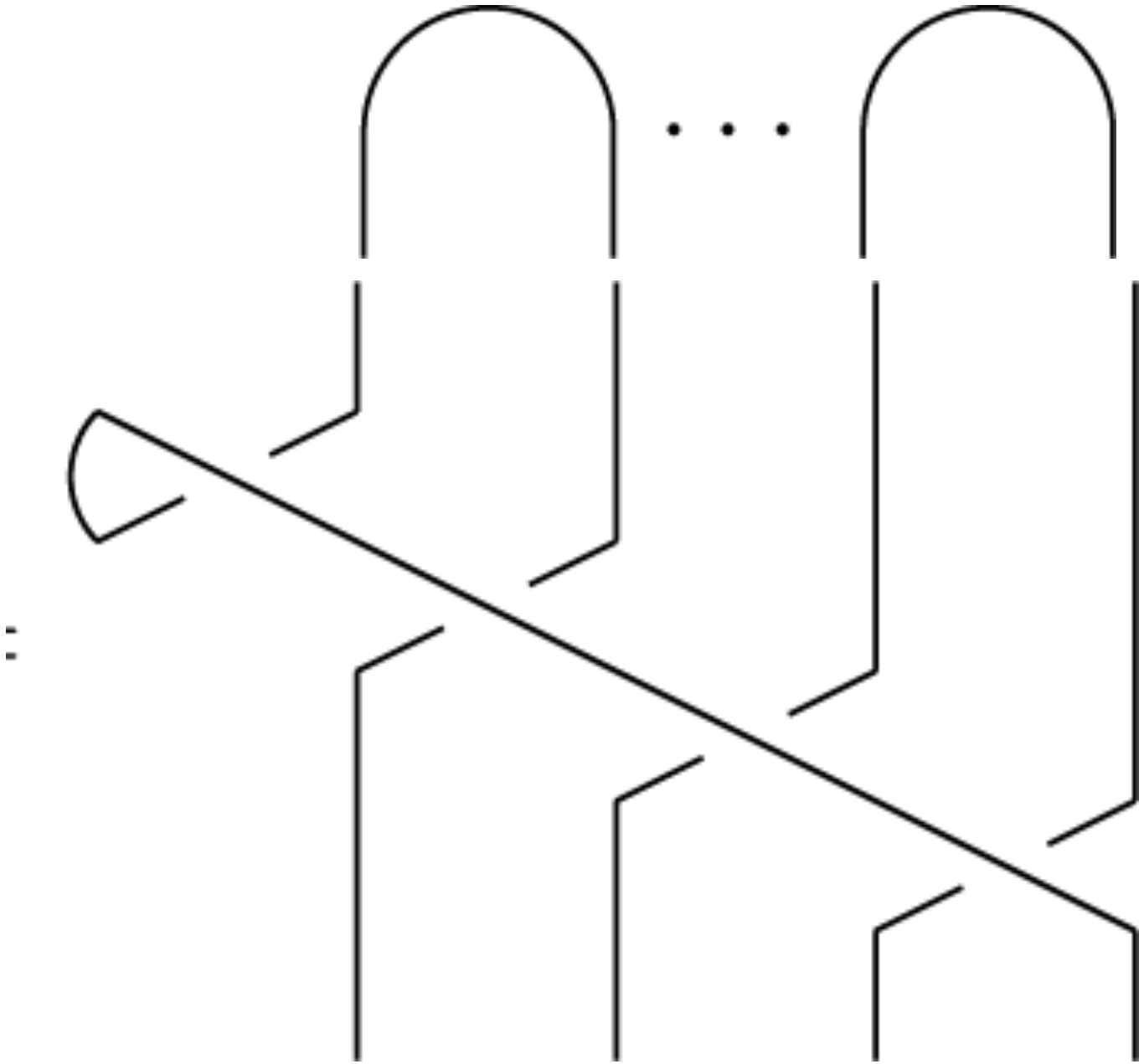
Ideas connect to Quantum Information

(with Zhengwei Liu and Alex Wozniakowski)

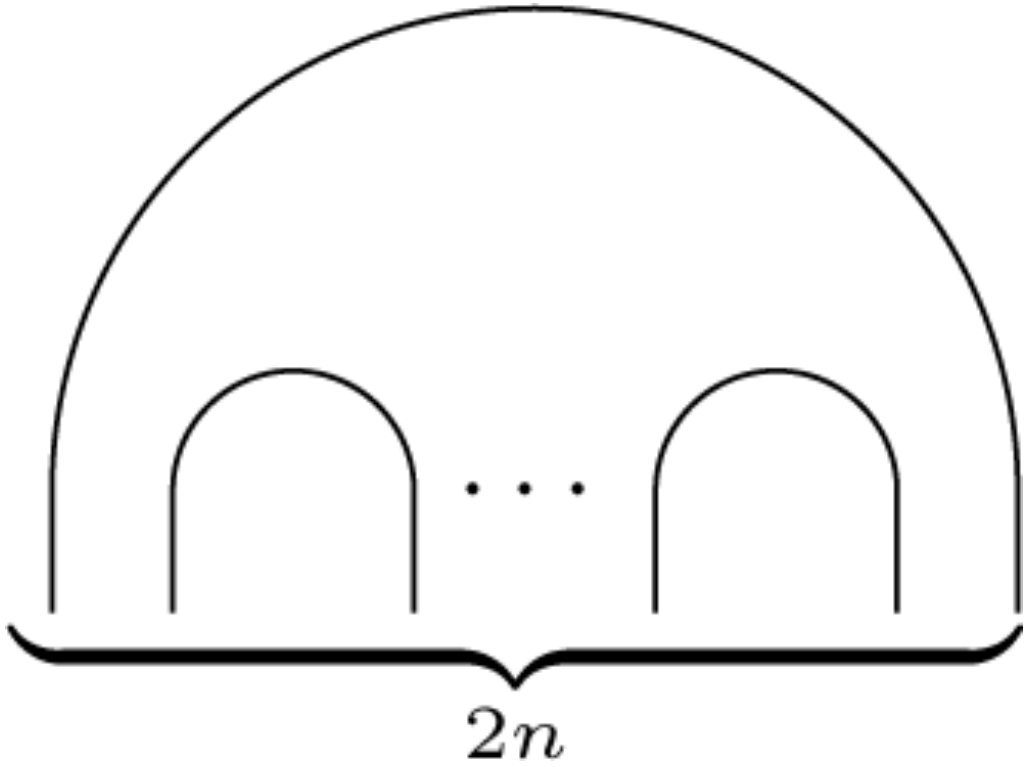
http://arxiv.org/abs/1605.00127
http://arxiv.org/abs/1605.00321

- Teleportation

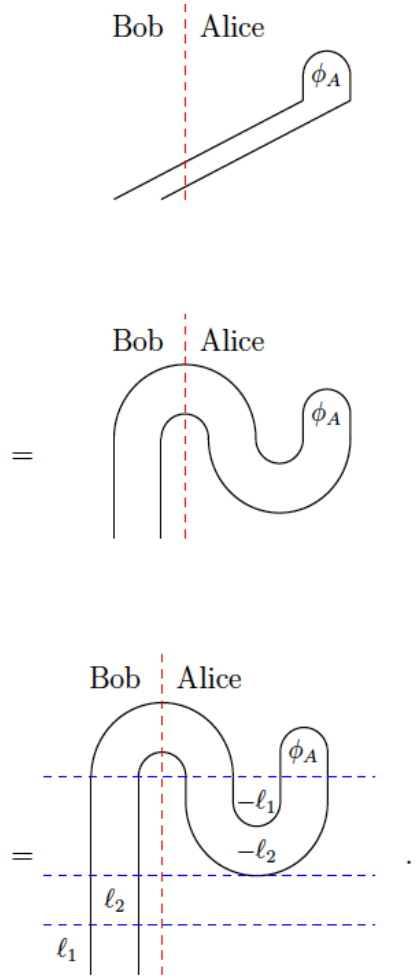


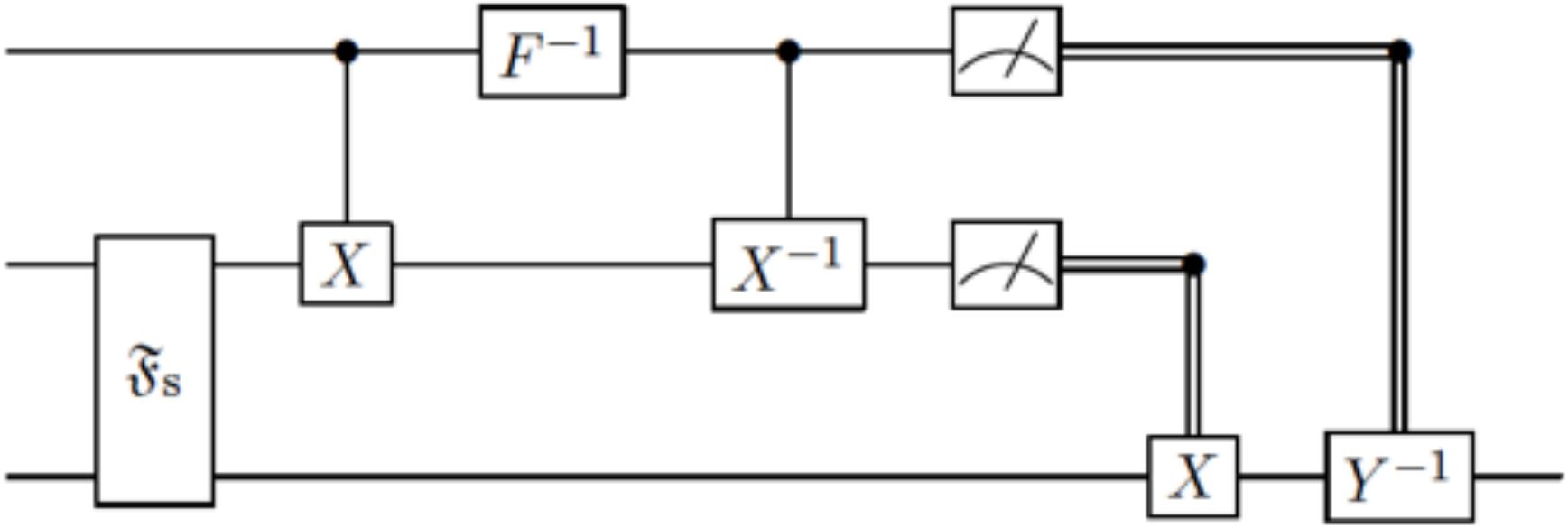

State
SFT

= Maximally Entangled Resource State



$$\mathfrak{F}_s|\vec{0}\,\rangle = |\mathrm{Max}\rangle$$


Bob
Alice
ϕ_A
=
Bob
Alice
ϕ_A
=
Bob
Alice
ϕ_A
$-\ell_1$
$-\ell_2$
ℓ_2
ℓ_1



Classic Teleportation Circuit of Charles Bennett et. al.

II. CT DETAILS

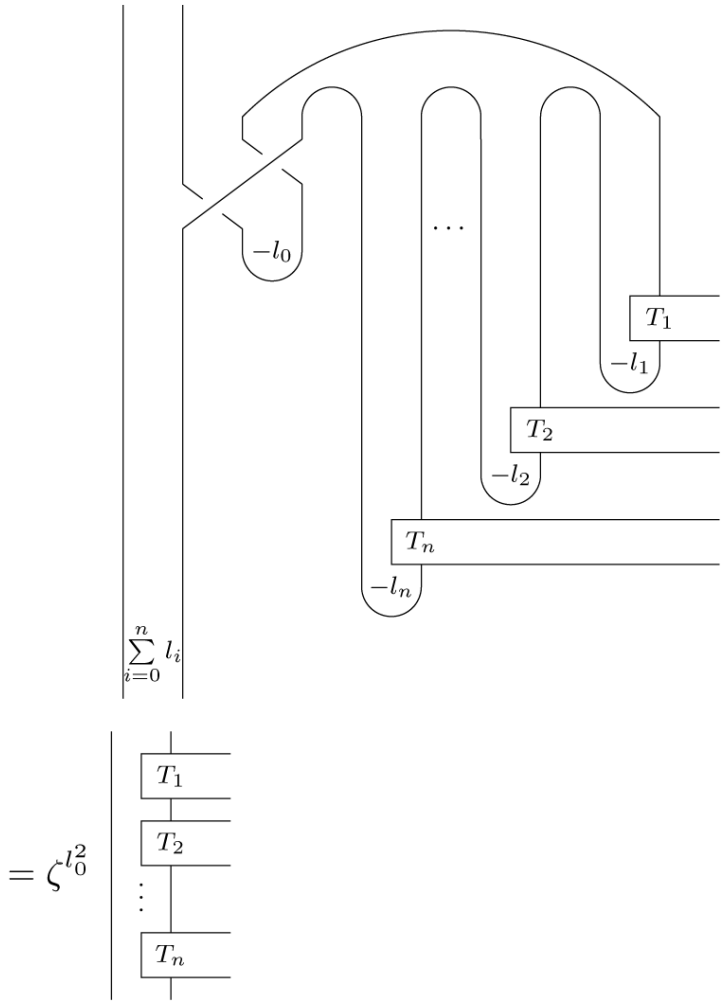


FIG. 3. Diagrammatic CT-protocol for X-compressed transformations.

A Further Reflection

Is the RP-C* relation helpful for understanding Robert's proof of RP in the context of Regge calculus?

And Now Some Photos!





Robert and Jim Glimm in Cargèse 1979



Konrad and Robert: 9 September 1992, White Hart Inn, Salisbury, CT



Robert and Roland Sénéor
Capri, May 1993
(courtesy Philippe Blanchard)





Robert at the celebration of Konrad 60.
ETH Aula June 2002



Robert with Shoucheng Zhang, his former diploma student in the 1980's
In Berlin in front of the Free University physics institute.



March 19, 2005 in my office at Harvard during the Coleman conference



With Ken Lane and Shelly Glashow



With Is Singer



Talking with Sidney and Diana Coleman



With Luboš Motl and Frank Wilczek



Robert’s Retirement 16 December 2005

19 May 2006
Berlin,
Dürers Mutter





27 February 2009 in Berlin before concert with Zehetmair & Berlin Philharmonic conducted by Heinz Holliger



With Erika 2009 before Concert in Berlin





Schloss Brunegg, May 14, 2016

February 28, 2009

Berlin Philharmonic conductor's room

Edicson Ruiz lesson from

Heinz Holliger on

"Preludio e Fuga für

Solokontrabass"





Skype call, May 23 2015



Erika’s home, Kahlendamm 3, Hannover



Hospice Luise, Hannover









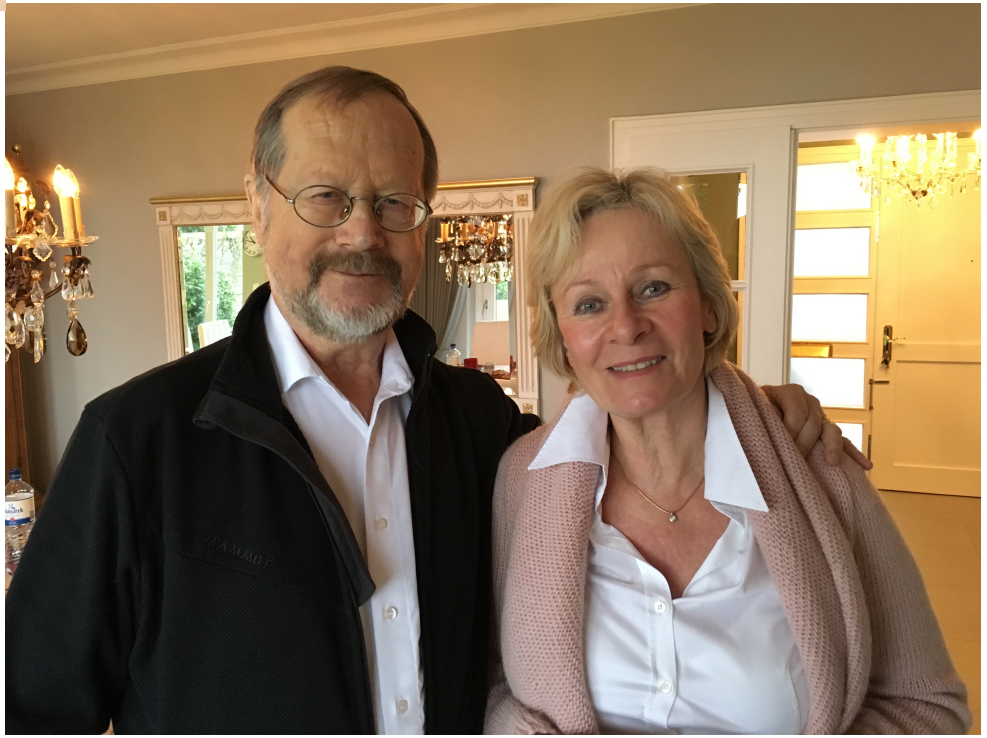






Ingrid and her daughter
Harald and his wife

February 5, 2016 Hamburg



Some Questions

1. Why is Osterwalder-Schrader positivity not a central feature in EVERY quantum physics textbook?
2. Why was Robert never a "full" professor?
3. Why did FUB not regard Robert as one of their most distinguished faculty members?
4. Can we understand Robert's Last Theorem? "Reflection Positivity in Simplicial Gravity," J. Phys. A, April 2016

Hopefully some progress will be made!



Many Thanks!!!