Multipartite Compressed Teleportation

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In a previous paper we introduced holographic software for quantum networks. This software suggests the definition of a compressed transformation. Here we utilize the software to find a lossless compressed teleportation protocol for multipartite communication for quantum networks. Our method realizes the minimal entanglement cost for teleportation.

I. INTRODUCTION

We begin with an explanation of the three concepts in the title.

A. Teleportation

Quantum communication had a significant advance when Bennett, Brassard, Crépeau, Jozsa, Peres, and Wooters introduced their teleportation protocol [1]. Preskill, Gottesman, and Chuang described the notion of quantum software for solving problems in quantum computation [2, 3]. Recently, Priandola and Braunstein cite teleportation as the "most promising mechanism for a future quantum internet" [4]. One can realize quantum networks through bidirectional quantum teleportation (BQST).

a. **Cost.** Zhou et al. and Eisert et al. pointed out that the cost of BQST is not optimized, and they gave a protocol to optimize the cost for certain 1-qudit transformations [5, 6]. This optimization has been further studied in [8], and in [9, 10] one finds extensive references.

Experimental work on long-distance teleportation has been achieved [11, 12]. The *Quantum Science Satellite* built by Pan and his coworkers provides an opportunity to test teleportation at record-breaking distances [13, 14].

Let us give an example to explain the idea of how compression can help. Suppose Alice and Bob want to share the transformation T, that acts on both Alice's qudit and Bob's qudit. Using the method of Bennett et al., Alice can teleport her input to Bob; then Bob can implement the transformation T locally on his computer and teleport the result back to Alice.

In this way, the cost in BQST is 2 entangled resource states (edits). However, if T is CNOT, one can realize this non-local transformation with the cost of only 1 edit.

So it is natural to ask the question: what transformations can be teleported with less cost, compared with BQST? In this paper, we give a topological description of such transformations, and we call them *topologically compressed*. This covers a large variety of transformations that are commonly used in protocols, and it also leads us to a new protocol.

B. Topological Compression

In our paper on holographic software (HS) [15], we give a topological simulation from our PAPPA model [16] to quantum information. This simulation captures topological features of fundamental concepts in communication, such as multipartite resource states, measurements, and the classical communication of measurement results.

Our diagrammatic approach in HS differs from previous diagrammatic approaches. One key point of departure is that in older work one represents a 1-qudit transformation by one string. But in our topological simulation, we represent a 1-qudit transformation by two charged strings.

This motivates us to study those special transformations that can be represented on a single string, as defined formally in (11). We call such transformations topologically compressed. We can teleport information that is captured on one string using 1 edit. In our picture, CNOT, the Tofolli gate, and controlled transformations are all topologically compressed.

a. **Compression Lowers Cost.** In §II we give a new protocol to teleport compressed transformations, that we call the CT protocol. This protocol relies on using local operations and classical communication (LOCC). It optimizes the entanglement resource cost for teleportation of transformations.

The CT protocol reduces the costs by 50% compared with BSQT. In BSQT, one needs two resource states. In our CT protocol, we use one resource state. Since CNOT, Tofolli, and controlled transformations are all topologically compressed, our protocol covers the previous teleportation protocols for CNOT, Tofolli, and controlled transformations.

C. Multipartite Communication

Greenberger, Horne, and Zeilinger introduced a multipartite resource state that we denote $|\mathrm{GHZ}\rangle$ in [18]. In HS we introduce our resource state $|\mathrm{Max}\rangle$, and we show that it is related to $|\mathrm{GHZ}\rangle$ by the quantum Fourier transform.

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a. **Multipartite Compressed Teleportation.** We show that the idea of topological compression is compatible with the topological feature of our multipartite resource state $|\mathrm{Max}\rangle$. Our CT protocol can be generalized to multipartite communication which costs one multipartite resource state. We call this generalization *multipartite compressed teleportation*. Our multipartite CT protocol does not reduce to multiple, bipartite communication. We give the details of the multipartite CT protocol (MCT) and describe an application of MCT. We do not give the details of the CT protocol in our paper, as one can figure it out as a special case of MCT.

If one realizes this teleportation by BQST, then one would need n bipartite resource states, and constructing these requires $2n$ noiseless channels. In our MCT protocol, we use one n-partite resource, which requires n noiseless channels to construct. Therefore we reduce the cost by 50%.

In this paper, we give our MTC in the usual algebraic terminology as a circuit in FIG. 1. From this picture, one can understand the protocol without knowing its topological significance. However, for additional insight, we also give its original topological simulation in FIG. 3. We refer the reader to [15] for the HS dictionary that translates between the two languages. One can also find out there the meaning of our diagrams that appear in FIG. 3.

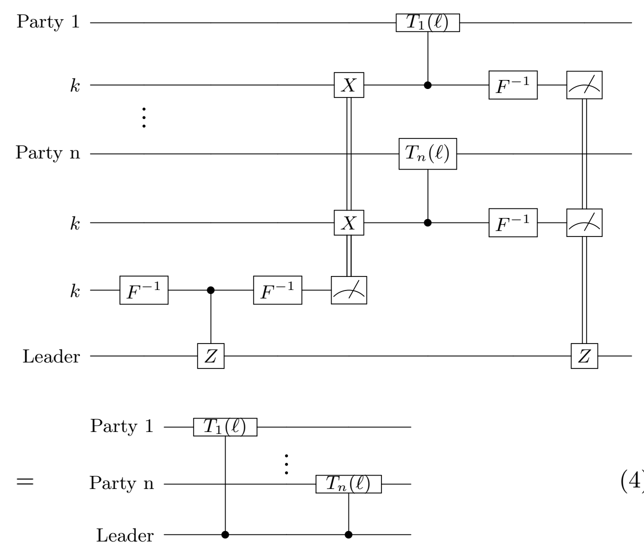


FIG. 1. The MCT algebraic protocol for controlled tranformations: The k's arise from $|\mathrm{GHZ}\rangle$ in (3) for the leader and the persons P_j, $1 \leqslant j \leqslant n$. The output of the protocol is the multipartite, controlled transformation T_c.

II. MULTIPARTITE COMMUNICATION

We consider 1-qudits with d possible values of the charge, considered as elements of Z_d. Let $|\vec{k}\rangle$ denote an n-qudit state with charges $\vec{k} = (k_1, \cdots, k_n)$, and let $|\vec{k}| = \sum_{j=1}^{n} k_j$. Here k_j, $|\vec{k}| \in Z_d$. In [15] we introduced the multipartite entangled resource state

$$|\mathrm{Max}\rangle = d^{\frac{1-n}{2}} \sum_{|\vec{k}|=0} |\vec{k}\rangle \ . \tag{1}$$

On the other hand, the $|\mathrm{GHZ}\rangle$ resource state is

$$|\mathrm{GHZ}\rangle = d^{-\frac{1}{2}} \sum_{l=0}^{d-1} |k, k, \cdots, k\rangle \ , \tag{2}$$

and experiments on $|\mathrm{GHZ}\rangle$ have been achieved [19, 20].

In fact these two resource states are related by the quantum Fourier transform, which is the local operation

$$|\mathrm{GHZ}\rangle = (F \otimes \cdots \otimes F)|\mathrm{Max}\rangle \ . \tag{3}$$

Here F denotes 1-qudit quantum Fourier transform defined in (8).

Let us describe our MCT protocol in FIG. 1. Suppose a network has one leader and n parties. Also assume that the j^{th} party can perform a controlled transformation

$$T_j = \sum_{l=0}^{d-1} |\ell\rangle\langle\ell| \otimes T_j(\ell), \tag{5}$$

where the control qudit belongs to the person P_j in the j^{th} party, and $T_j(\ell)$ can be an arbitrary multi-person, multi-qudit transformation on the targets. (The protocol of the controlled transformation T_j is shown in FIG. 2.)

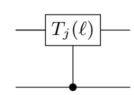


FIG. 2. Controlled transformations.

With these assumptions, the leader can perform a controlled transformation to all parties in the network,

$$T_c = \sum_{l=0}^{d-1} |\ell\rangle\langle\ell| \otimes T_n(\ell) \otimes \cdots \otimes T_1(\ell) \ . \tag{6}$$

This requires using a multipartite resource state $|\mathrm{Max}\rangle$ among the leader and the persons P_j. The leader has the common control qudit in T, and the j^{th} party performs the transformation $T_{j,l}$ for control qudit ℓ. This protocol costs one resource state $|\mathrm{Max}\rangle$ and $2n$ cdits. The time cost is the transmission of two cdits and the implementation of local transformations.

In [15] we analyze the BVK protocol [17] using holographic software. It would be interesting to analyze other protocols by this method, such as those in [3, 7, 21–28].

III. MCT DETAILS

Let $q^d = 1$ and $\zeta = q^{1/2}$ be a square root of q with the property $\zeta^{d^2} = 1$. Matrices X, Y, Z, F, G play an important role. Three of these are the qudit Pauli matrices

$$X|k\rangle = |k+1\rangle \ , \ Y|k\rangle = \zeta^{1-2k}|k-1\rangle \ , \ Z|k\rangle = q^k|k\rangle \ . \quad (7)$$

The quantum Fourier transform matrix F and the Gaussian G are

$$F|k\rangle = \frac{1}{\sqrt{d}} \sum_{\ell=0}^{d-1} q^{k\ell}|\ell\rangle \ , \quad G|k\rangle = \zeta^{k^2}|k\rangle \ . \quad (8)$$

These matrices satisfy the relations

$$XY = qYX \ , \quad YZ = qZY \ , \quad ZX = qXZ \ , \quad (9)$$

$$XYZ = \zeta \ , \quad FXF^{-1} = Z \ , \quad GXG^{-1} = Y^{-1} \ . \quad (10)$$

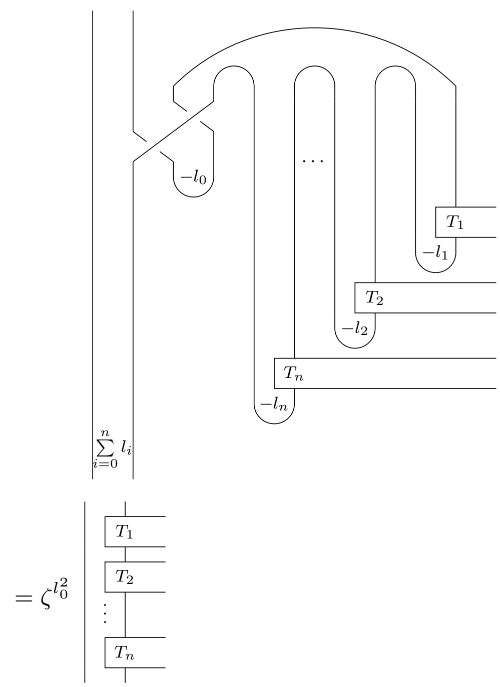


FIG. 3. Diagrammatic MCT-protocol for X-compressed transformations.

a. **Definition of Compressed Transformations.** We define compressed transformations in an algebraic manner. We say that a transformation T is Z-compressed on the i^{th}-qudit if T commutes with the action of Pauli Z on the i^{th}-qudit, and arbitrary transformations on the other qudits.

We say that a transformation T is Z-compressed on the first qudit if T is

$$T = \sum_{\ell=0}^{d-1} |\ell\rangle\langle\ell| \otimes T(\ell) \ . \quad (11)$$

Note that T can also be considered as a controlled transformation, or a block diagonal transformation.

We also have the following equivalent condition: T commutes with the action of Pauli Z on the first qudit. We say T is Z-compressed on the i^{th}-qudit, if it commutes with the action of Pauli Z on the i^{th} qudit.

Similarly we say T is X (or Y)-compressed on the i^{th} qudit, if it commutes with the action of Pauli X (or Y) on the i^{th} qudit. We can switch between the three compressed transformations using $FXF^{-1} = Z$ and $GXG^{-1} = Y^{-1}$; see §II B of [15] for details.

We extend the notion by unitary equivalence. We say that a transformation T' is compressed on the i^{th}-qudit if $T' = UTV$, where T is Z-compressed on the i^{th}-qudit and U, V are local transformations on the i^{th}-qudit.

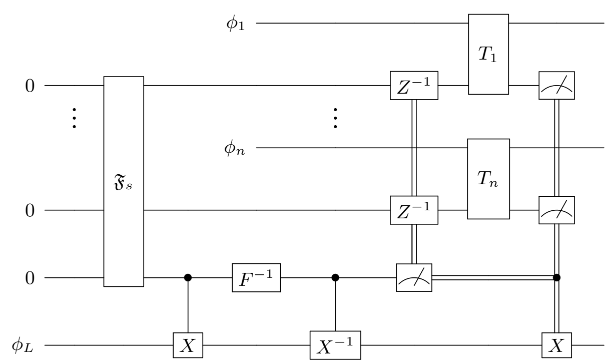
b. **MCT Protocol.** We give the MCT diagrammatic protocol for X-compressed transformations in FIG. 3.

Using our dictionary HS of holographic software, we give the MCT algebraic protocol in FIG. 4 corresponding to the diagrammatic protocol in FIG. 3. Here we use a key transformation that we name the *string Fourier transform* $\mathfrak{F}_s$. We use $\mathfrak{F}_s$ to provide a maximally entangled state from a product state.

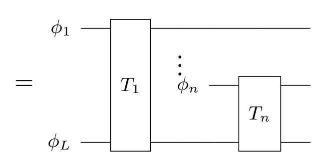
We show in HS that our resource state is

$$|\text{Max}\rangle = \mathfrak{F}_s|\vec{0}\rangle \ , \quad (12)$$

which replaces $|\text{Max}\rangle = (F^{-1} \otimes \cdots \otimes F^{-1})|\text{GHZ}\rangle$ given by (3). Here $|\vec{0}\rangle$ denotes the n-qudit with charge 0 for each 1-qudit. We call $|\vec{0}\rangle$ the ground state; it plays the role of ancilla qudits in our protocol. This explains our using the resource input in FIG. 4 as $\mathfrak{F}_s|\vec{0}\rangle$.



(13)



(14)

FIG. 4. MCT protocol for X compressed tranformations: The resource state $|\text{Max}\rangle$ is expressed as $\mathfrak{F}_s|\vec{0}\rangle$. One can simplify the protocol by the identity in FIG. 5

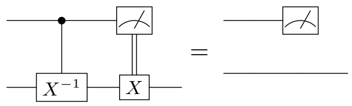


FIG. 5.

Taking the conjugation of local transformations, we obtain the MCT protocol for compressed transformations. In particular, taking the conjugate of the Fourier transform F, we obtain the MCT protocol for Z-compressed transformations (or multipartite, controlled transformations) in FIG. 1.

In the case with only two persons, the MCT protocol says: Assume that a quantum network can perform a transformation T, which is compressed on a 1-qudit belonging to a network member Alice. Then Alice can teleport her 1-qudit transformation to Bob using one edit and two cdits. One can easily derive the entanglement-swapping protocol, and the teleportation of the Tofolli gate from it.

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